Generalizing Cayley Maps

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All five Platonic solids are orientably regular maps.

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The automorphism group of a regular orientable map acts transitively on the set of darts of the map, and the stabilizer of any vertex of a regular map is cyclic of order equal to the valency of the map.

Constructing Regular Maps

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But we will not take this rout.

Monodromy group

- if λ denotes the **arc-reversing** permutation of the dart set mapping a dart e to its opposite dart e^{-1} and ρ is the permutation of the dart set defined as the union of the cycles ρ_v, then the **monodromy group** of $\mathcal{M}$ is the group $\langle \lambda, \rho \rangle$

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Theorem

$$|\langle \lambda, \rho \rangle| \geq |D(\mathcal{M})|$$

The map $\mathcal{M} = (\Gamma, \rho)$ is regular if and only if $|\langle \lambda, \rho \rangle| = |D(\mathcal{M}|$.

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- the local permutations ρ_v need to be selected so that all the automorphisms in G **lift**, i.e., become map automorphisms, i.e., commute with ρ

Characterization of Graphs That Admit Regular Embeddings

Theorem (Surowski; Gardiner, Nedela, Širáň, Škoviera)

A connected graph Γ *of valency greater than or equal to* 3 *admits an embedding as an orientably-regular map (on some closed orientable surface) if and only if its automorphism group contains a subgroup* G *acting transitively on* $D(\Gamma)$ *and such that the stabilizer* G_v *of every vertex is cyclic.*

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- each automorphism $\varphi \in G$ *lifts* to an automorphism of (Γ, ρ)
- the embedding is independent of the choice of the μ_v's

Corollary

If a graph Γ *is vertex-transitive and* $\varphi \in Aut(\Gamma)$ *fixes at least one vertex of* Γ*, there exists an embedding* $\mathcal{M} = (\Gamma, \rho)$ *such that* φ *lifts into* $Aut(\mathcal{M})$*.*

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If one wants to start from a graph, which will then be embedded as a regular map, then one **has** to start from an arc-transitive graph with an automorphism group G described in the Surowski; Gardiner, Nedela, Širáň, Škoviera Theorem.

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- if, in addition, if there exists a **group automorphism** φ of G that preserves X and acts cyclically on X, then choosing $\rho(g,x)=(g,\varphi(x))$ gives rise to a **regular map**

In 1990's, Širáň and Škoviera (1990's) generalized Biggs' result by observing that the orbit of the φ has to satisfy the identity $p(x^{-1}) = p(x)^{-1}$, named such maps **balanced Cayley maps**, and proved:

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Theorem (Škoviera, Širáň)

Let $\mathcal{M} = CM(G, X, p)$ be a balanced Cayley map. Then $\mathcal{M}$ is regular iff there exists a group automorphism φ of G satisfying the property $\varphi(x) = p(x)$ for all $x \in X$.

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For example, all regular embeddings of the complete graphs K_n turn out to be balanced Cayley maps (James, Jones, 1985).

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Four of the five (regular) Platonic solids are Cayley maps, three are balanced.

Basic observations:

$$|Aut(CM(G, X, p))| \leq |G| \cdot |X|$$

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there exists a $\varphi \in Aut(CM(G,X,p))$ such that
$\varphi(1_G) = 1_G$ and $\varphi((1_G,x)) = (1_G,p(x))$

Definition (RJ,Širáň)

A *skew-morphism* of a group G is a permutation φ of G preserving the identity and satisfying the property

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for all $g, h \in G$ and a function $\pi : G \to \mathbb{Z}_{|\varphi|}$, called the *power function* of G.

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Let $\mathcal{M} = CM(G, X, p)$ be any Cayley map. Then $\mathcal{M}$ is regular iff there exists a skew-morphism φ of G satisfying the property $\varphi(x) = p(x)$ for all $x \in X$.

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- (some people believe it might be equal to 1)

Lifting the underlying Cayley group

Theorem (Richter, Širáň, RJ, Tucker, Watkins)

An embedding $\mathcal{M}$ of a Cayley graph $C(G, X)$ admits lifting each of the left-regular multiplications by the elements of G into automorphisms of $\mathcal{M}$ if and only if $\mathcal{M}$ is a Cayley map, i.e., the local rotation at each vertex is the same.

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As a *consequence*, if we move away from Cayley maps, there will be some left-multiplication automorphisms of the underlying graph that will not lift, and the map might not end up being vertex-transitive.

Theorem

Let H be a subgroup of a finite group G. Then the group of graph automorphisms induced by left multiplications via the elements of H lifts into a group of automorphisms of a map $\mathcal{M} = (C(G,X),\rho)$ if and only if ρ is constant on the right cosets of H in G,

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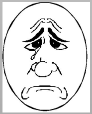
The (vertex) orbits of the group of automorphisms obtained by lifting left multiplications by the elements of H are the right cosets of H in G.



Lifting automorphisms that fix a vertex

Theorem

Let φ be a graph automorphism of a Cayley graph $C(G, X)$ that fixes the identity, $\varphi(1_G) = 1_G$. Then φ lifts into a map automorphism of a map $\mathcal{M} = (G, X, \rho)$ if and only if

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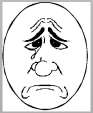
Skew-morphisms lift if and only if ρ is constant on G.



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- there exist automorphisms mapping 1_G to all the different right cosets of H in G
- special situation occurs when φ has only few orbits on the right cosets of H in G

The proportion of Cayley maps among orientably regular maps

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- (if almost all vertex-transitive graphs are Cayley, then maybe almost all orientably regular maps are also Cayley)

Спасибо!